An Algebraic Adventure with The Rubik's® Cube

How to Solve the Rubik's Cube
Secondary Curriculum Workbook
Teacher Guide
Let's get ready to journey together on an adventure using the Rubik's Cube. In this workbook, students will not only learn to solve the Rubik's Cube, but they will tackle long-standing questions about how the cube behaves, how it's made, and why the common attempt to solve the cube one color at a time does not work.

The workbook has been created with independent challenges in mind. Whether you choose to use this book as a whole-class project, small group project, or as an independent study for select students, the concepts needed to challenge each level of learner have been embedded. Many of the questions and leading statements are quite open-ended, leaving you the opportunity to incorporate any level of algebra you wish. In this Teacher Guide, "Teacher Notes" have been included throughout the workbook with possible Common Core related ideas. These ideas are merely suggestions and only touch the surface of possibilities available. Common Core Mathematical Practices and Standards have been noted as well. Again, these only touch the surface of the Standards applicability. The possibilities are endless!

In addition to Mathematical Practices and Standards, this book was designed using the Common Core Modeling Cycle as presented in the Common Core Standards Initiative for Mathematics document under High School-Modeling, and is explained in the document as a process to link classroom mathematics to everyday life, work, and decision-making.

The basic modeling cycle is summarized in the diagram. It involves:

1. Identifying variables in the situation and selecting those that represent essential features,
2. Formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables,
3. Analyzing and performing operations on these relationships to draw conclusions,
4. Interpreting the results of the mathematics in terms of the original situation,
5. Validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable,
6. Reporting on the conclusions and the reasoning behind them.

Choices, assumptions, and approximations are present throughout this cycle.

The following is a list of objectives from The Common Core Standards for Mathematics Algebra Overview that this workbook is designed to support:

- Create equations that describe numbers or relationships.
- Understand solving equations as a process of reasoning and explaining reasoning.

We hope you and your students enjoy deciphering the algorithms presented in the Solution Guide and using algebraic concepts to create a more efficient way to solve the Rubik's Cube. The mathematical knowledge required and learned using this manipulative provides a plethora of direct and indirect educational opportunities.
Welcome to your Algebraic Adventure with The Rubik’s Cube!

To complete this workbook, you will need self-motivation, a desire for personal achievement, the ability to think outside the box, and sufficient knowledge of algebra with regards to like terms, distributive property, inverse property, variables, algorithms, expressions, and equations.

This workbook has three parts:

• Part 1 - Solving the Rubik’s Cube.
• Part 2a - Collecting observational data about the algorithms used to solve the Rubik’s Cube.
• Part 2b - Redacting and combining terms in the algorithms to create more efficient algorithms.
• Part 3 - Creating a revised Solution Guide using the algorithms developed in Part 2b.

Although Part 2 has been separated into two sections, you may find it easier to work on both sections simultaneously.

The gaming/puzzle nature of the Rubik’s Cube will allow you to practice your math skills in a non-math environment. Within the pages of this workbook, you will be challenging your ability:

• to create sequences that describe relationships between faces of the cube and independent pieces within the cube.
• to understand processes of reasoning.
• to understand processes of explaining reasoning.
• to persevere in solving problematic situations.
• to make good use of structural facts.

Ultimately, by redacting and combining pieces of each algorithm, you will be able to relate and apply your observations and accomplishments to numerical and other algebraic relationships. This workbook focuses on relating algebraic ideas to the Rubik’s Cube through the Common Core Mathematical Practices and Modeling Cycle for problem solving.

The Common Core Standards and Practices Modeling Cycle for problem solving is shown below:

![Common Core Modeling Cycle]

This Modeling Cycle will be used throughout the workbook to help you manage your observations and reasoning patterns.

Common Core Standards:

- CCCS.Math.Content.HSA-CED.A1
- CCCS.Math.Content.HSA-CED.A2
- CCCS.Math.Content.HSA-CED.A4
- CCCS.Math.Content.HSA-REI-A1
- CCCS.Math.Practice.MP1
- CCCS.Math.Practice.MP3
- CCCS.Math.Practice.MP7
- CCCS.Math.Practice.MP8

Teachers:

The abstract process of creating and dissecting the sequences/algorithms will help students to train their minds in step-by-step problem solving methods. In addition, the students will begin to identify with necessary steps, such as stepping back to look at the big picture and remembering the end goal, without getting distracted by the immediate solutions in between each step. While student knowledge of Algebra is recommended, it is not an absolute requirement as the Rubik’s Cube will provide additional understanding of broader concepts allowing you to teach with primarily top-down processing in mind.

Teachers:

This book has been designed to be used as an all-encompassing independent assignment or in pieces as given lessons at your discretion. Regardless of the way you choose to use this book in your classroom, it is imperative that the students do not skip around. Some students may already know how to solve the Rubik’s Cube. It is important that those students complete the Part 1 questions to prepare their minds for the thought processes necessary in Parts 2a and 2b. If you, or your students, wish to complete Parts 2a and 2b simultaneously, it is recommended that both sections be thoroughly read before beginning.
An Algebraic Adventure - Teacher Guide

Part 1

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The first part, solving the Rubik’s Cube, is intended to be an independent study; however, working with a partner may increase your understanding as you work through difficult stages.

Let’s begin...

Start by getting familiar with your cube. Please refer to Stage 1 of the Solution Guide.

Notice the colors and positions of the colors of the cube.

Which colors are opposite each other?

WHITE and YELLOW
BLUE and GREEN
RED and ORANGE

This will always be true. The center piece of each face will always be the center piece for that color. It does not move.

Note: The difference between Layer and Face. “Layers” are three-dimensional (length, width and height). The word “face” refers to the flat area of a “layer.” Faces are two-dimensional (length and width).

Draw an arrow on the cubes below to demonstrate the quarter turn moves that relate to the letter terms:

Teachers:
A variety of pairings will work with Part 1. Students may be paired by mathematics level, Rubik’s Cube solving ability, or simply by self-choice.

Teachers:
The answers provided in this workbook are sample answers and by no means should they be considered the only answers. Students will develop their own responses. The Rubik’s Cube has 43 quintillion permutations so there are many possible correct answers.
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Now, let's try some practice moves so you can familiarize yourself with the terms for each face rotation. If you are familiar with solving the cube, you may proceed to Page 4.

Start with a solved cube. If your cube is unsolved you may wish to use removable identification (color stickers or labels with letter coding, for example Y=Yellow, R=Red, etc.) and pretend the cube is solved. Upon completion of this section, the stickers should be removed.

Complete the following moves, in order. Remember to keep the same face facing you for the entire set of algorithms. You should be familiar with the letter terms after completing the previous exercise. If not refer to Page 2 in the Solution Guide for the quarter turn moves.

1

To create a Multi-colored Cross

| R | R | L | L | U | U | D | D | F | F | B | B |

Return to a solved cube

| Bi | Bi | Fi | Fi | Di | Di | Ui | Ui | Li | Li | Li | Ri | Ri |

To create a Square in the Middle

| R | Li | U | Di | Fi | B | R | Li |

Return to a solved cube

| L | Ri | Bi | F | D | Ui | L | Ri |

In the space below, make some notes for yourself, if needed.

- performing the algorithm backwards & inverted reverses the algorithm.

Teachers:

The icons that represent each quarter turn move have been eliminated from this workbook to encourage students to use the term IDs as rules to facilitate their learning, so they do not become dependent on the pictures as guides.

However, should a student need modifications, the Solution Guide, with the pictorial guide, can be used.

In either case, students will be working to accomplish the expectations set forth in CCSS Algebra, Seeing Structure in Expressions Standard 1a, (interpreting parts of an expression, such as terms, factors, and coefficients) as they work through this workbook.

Teachers:

If students need additional space to complete answers, instruct them to complete on separate piece of paper and staple to the appropriate page.
Now that you are familiar with the Rubik’s Cube terms and behaviors, it’s time to solve the cube.

If your cube is in the solved state, you need it to be in an unsolved state. An unsolved cube is considered to be in a scrambled state. For the purpose of this adventure, you will use ANY twenty-five quarter turns to create a scrambled state.

NOTE: For color pictures, see the SOLUTION GUIDE.

Once you have scrambled your cube, follow the steps in STAGE 2 of the SOLUTION GUIDE to get the WHITE CROSS.

GOAL:

Practice this stage several times before proceeding to the next stage. It is not necessary for you to memorize the algorithm as you can refer to the SOLUTION GUIDE, as needed.

In the space below, note any observations you make about the algorithms.

- If an edge piece is directly under the position it should be in, with the white on the down face and the color matching the center piece is facing you, the algorithm is not needed; only R, R, (or Ri, Ri) is needed to position the white edge in its correct place.

- The last move, Ui, is always needed to put the pattern in its correct position.
In the next stage, you will complete the **WHITE FACE** by getting the white corners in their correct places. Follow the steps in **Stage 3** of the **Solution Guide**.

**GOAL:**

![Diagram of a Rubik's cube with white corners highlighted.](image)

- Must match center piece color

**TIP:**

If you accidentally place a WHITE corner with the incorrect face, reposition the WHITE corner on the DOWN face under its intended position on the UP face and use the algorithm repeatedly (as many times as it takes) to get the WHITE of the corner piece on the UP face.

<table>
<thead>
<tr>
<th>Ri</th>
<th>Di</th>
<th>R</th>
<th>D</th>
</tr>
</thead>
</table>

Make sure the WHITE cross remains intact each time you reposition a WHITE corner.

Now scramble your cube and try **Stage 2** and **Stage 3** several times together.

In the space below, note any observations you make about the algorithms. In Part 2, you will take a closer look at these two algorithms for possible “like moves” (think: like terms) that can be combined or redacted for a more efficient algorithm.

- The last move is not always needed. Sometimes, if the last move is skipped, the next corner is already in place directly under where it should be.

- Depending on where the white face of the corner piece is, you can figure out how many times you have to do the algorithm to get the corner piece in the correct position.

  - If white is on the right face = 1x
  - If white is on the down face = 3x
  - If white is on the front face = 5x.

- When you decide which white corner to solve first, it’s more efficient to try and solve the corners where the white is on the right face.

- The only moves used in this stage are Ri, Di, R, D.

**Teachers:**

In this particular stage, **STAGE 3**, the concept of truncating zeros, is also applicable. The last “move” in the algorithm for **STAGE 3** is often unnecessary, and therefore arbitrary if done.

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**An Algebraic Adventure**

**TIP:**

If you accidentally place a WHITE corner with the incorrect face, reposition the WHITE corner on the DOWN face under its intended position on the UP face and use the algorithm repeatedly (as many times as it takes) to get the WHITE of the corner piece on the UP face.

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**Part 1**

Teachers: In this particular stage, **STAGE 3**, the concept of truncating zeros, is also applicable. The last “move” in the algorithm for **STAGE 3** is often unnecessary, and therefore arbitrary if done.
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You are halfway to a solved Rubik's Cube! Take a deep breath and remember, part of your goal in completing this adventure is to persevere when difficult algorithmic tasks present themselves.

In this stage, pay close attention to the movement and placement of each edge piece. You may make some interesting observations that can be helpful in Part 2. Follow the steps in Stage 4 of the Solution Guide to complete the MIDDLE LAYER.

GOAL:

Must match center piece color

TIP:

If you accidentally move an edge piece to the wrong place, leave it there and continue working with the other edge pieces. Eventually, you will find the correct edge piece and place it in its correct position. The misplaced edge piece will be moved and available to move to its correct place.

Clockwise: 

| U | R | Ui | Ri | Ui | Fi | U | F |

Counter-Clockwise:

| Ui | Li | U | L | U | F | Ui | Fi |

Did you notice anything about the edge piece transformations? Use the space below to note your observations.

- The algorithm always starts in the opposite direction of where you want to move the face of middle piece on the top layer.
- If there's only YELLOW faces to work with, you can do one of the algorithms to temporarily move it to the middle layer so you can free up another piece.

A common that you may encounter is not having a non-yellow edge. When this happens, there is a temporary solution. Should you encounter this during this stage, observe and work with your cube to a solution. Explain your solution below using both mathematical and Rubik's Cube terms:

If there is no non-YELLOW edge pieces to work with, you can locate an edge on the middle layer that is in the wrong place and use an algorithm to place the non-YELLOW edge where the misplaced middle layer edge is located. Basically, you are substituting a non-YELLOW edge for the usable edge and swapping their positions. Then, you can place that misplaced edge in its correct position using one of the given algorithms.

Note: Though you are using the Modeling Cycle throughout the entirety of this workbook, you can also use the Modeling Cycle to solve a simpler within your whole project.

Teachers:

This concept of rewriting expressions, simplifying, performing a seemingly unrelated calculation in order to move forward with a solution is quite prevalent in STAGE 4. Often, students will find that they must complete a seemingly unnecessary step in order to place the cube in a position that allows them to advance to the next step in this stage.
This next stage has two parts. Follow the steps in Stage 5 of the Solution Guide.

First, you will complete the YELLOW CROSS, then the YELLOW CORNERS. Pay close attention to the images in the Solution Guide. The steps in this stage rely on matching the image on the YELLOW face of your cube with an image on the YELLOW face of the cubes shown in the Solution Guide.

**GOAL: Stage 5a**

- TIP: If you are unsure about the image on your cube, or if your cube doesn’t match one of the three options exactly, find the best match. For example, if your YELLOW face has a YELLOW center piece and a YELLOW edge, the most accurate match would be the single YELLOW center piece.

This may be the trickiest part of the solution. Don’t worry; you can do it!

**GOAL: Stage 5b**

- TIP: If you are unsure about the image on your cube, or if your cube doesn’t match one of the three options exactly, you might need to rotate (turn without twisting any layers) your cube.

In the space below, note any observations you make about the algorithms.

- If you have to repeat the algorithm and **DO NOT** have to rotate the cube to match an UP face image, you can skip the last move and the first move and just do U, U, like this: R, U, Ri, U, R, U, U, then U, Ri, U, R, U, U.
Part 1

Make sure the YELLOW face is the UP face.

We will focus on the LEFT/FRONT/UP corner piece.

Match your cube to one of the following images:

- If no YELLOW corner pieces are on the UP face:
  - The LEFT/FRONT/UP corner has a YELLOW piece on the LEFT face.

- If one YELLOW corner piece is on the UP face:
  - The LEFT/FRONT/UP corner has a YELLOW piece on the UP face.

- If any two YELLOW corner pieces are on the UP face in any position:
  - The LEFT/FRONT/UP corner has a YELLOW piece on the FRONT face.

Before you move on to the last solving stage, take a look at your cube.

- Does the DOWN face have a completed WHITE face?
- Does the bottom layer and middle layer of each face match with the color of the center piece?
- Is there a YELLOW cross on the UP face?

Note: At this point, the colors on the corner pieces on the UP face do not matter. Some may be YELLOW, but may not be in the correct place.

Did you notice any similarities or differences to solving the WHITE cross and the YELLOW cross? Use the space below to note your observations.

- The white cross was very systematic. The solution was based on matching a pattern on the vertical middle layer.

- The white cross required you to use the bottom layer to position the white edges below where they were supposed to go.

- The yellow cross doesn't use the bottom layer. The yellow cross uses manipulations of the top layer to get the yellow cross.

Teachers:

Another way to describe the position of the corner is the “corner at the bottom, right-hand of the cube.” This corner will always be part of the FRONT face, RIGHT face and DOWN face.
The Last Stage!

Follow the steps in Stage 6 of the Solution Guide to complete the YELLOW FACE.

Stage 6 is also separated into two parts. In part one, you will complete the YELLOW FACE by positioning the YELLOW CORNERS and in part two, you will reposition the YELLOW EDGES on the UP face so the top layer of all the side faces is completely solved.

**GOAL: Stage 6a**

| Ri | F | Ri | B | B | R | Fi | Ri | B | B | R | R | Ui |

**TIP:**

If you cannot find matching pieces on the same face or the diagonal, do the algorithm until you do have matching horizontal corners or matching diagonal corners.

Be sure to check for matches after every completion of the algorithm.

**GOAL: Stage 6b**

| Clockwise: F | F | U | L | Ri | F | F | Li | R | U | F | F |

| Counter-Clockwise: F | F | Ui | L | Ri | F | F | Li | R | Ui | F | F |

**TIP:**

Occasionally, solving the YELLOW edges will not solve the cube completely. It may solve each of the faces of each layer, but the top layer may not match the middle layer or bottom layer. If the top layer does not match the other layers, twist the top layer until all faces are complete.

Congratulations! You **DID IT!** You SOLVED the Rubik’s Cube!
Did you notice any similarities or differences to solving the WHITE face and the YELLOW face? Perhaps, you noticed some differences between putting the WHITE corners and YELLOW corners in the correct place.

Use the space below to note your observations.

- The white corners require repetition of the algorithm to position the white corners correctly.
- The white corners are placed individually.
- The yellow corners are placed simultaneously.

Now scramble your cube and solve it from the beginning.

Here are all of the algorithms in one place:

### WHITE Cross:

\[
\begin{align*}
R & \quad R \\
\rightarrow & \quad Ri \ U \ Fi \ Ui
\end{align*}
\]

### WHITE Corners:

\[
\begin{align*}
Ri & \quad Di \quad R \\
\rightarrow & \quad Ri \ Di \quad R \quad D
\end{align*}
\]

### MIDDLE Layer:

- **Clockwise:**
  \[
  \begin{align*}
  U & \quad R \quad Ui \quad Ri \quad Ui \quad Fi \quad U \quad F
  \end{align*}
  \]

- **Counter-Clockwise:**
  \[
  \begin{align*}
  Ui & \quad Li \quad U \quad L \quad U \quad F \quad Ui \quad Fi
  \end{align*}
  \]

### YELLOW Cross:

\[
\begin{align*}
F & \quad U \quad R \quad Ui \quad Ri \quad Fi
\end{align*}
\]

Or
\[
\begin{align*}
F & \quad R \quad U \quad Ri \quad Ui \quad Fi
\end{align*}
\]

### YELLOW Face:

\[
\begin{align*}
R & \quad U \quad Ri \quad U \quad R \quad U \quad U \quad Ri
\end{align*}
\]

### YELLOW Corners:

\[
\begin{align*}
Ri & \quad F \quad Ri \quad B \quad B \quad R \quad Fi \quad Ri \quad B \quad B \quad R \quad R \quad Ui
\end{align*}
\]

### YELLOW Edges:

- **Clockwise:**
  \[
  \begin{align*}
  F & \quad F \quad U \quad L \quad Ri \quad F \quad F \quad Li \quad R \quad U \quad F \quad F
  \end{align*}
  \]

- **Counter-Clockwise:**
  \[
  \begin{align*}
  F & \quad F \quad Ui \quad L \quad Ri \quad F \quad F \quad Li \quad R \quad Ui \quad F \quad F
  \end{align*}
  \]
Part 1

Before continuing to Part 2a, take a moment to make the following connections between Rubik’s terms and algebraic terms.

Webster’s Dictionary defines an algorithm as a process or set of rules to be followed in calculations or problem-solving operations.

What is an algorithm as it relates to algebra?  Give an example. What is an algorithm as it relates to the Rubik’s Cube?  Give an example.

An algorithm is a sequence of terms that lead to a desired outcome.  Ri, Di, R, D.

What is a term as it relates to Algebra?  Give an example.

A term is a quantity or representation of a quantity (e.g. a variable) in a expression or equation.  6x 2y.

What is a term as it relates to the Rubik’s Cube?  Give an example.

A term is the name/ID of a 1/4 turn move in a sequence such as R, Ri.

R  Ri  L  Li  B  Bi  D  Di  F  Fi  U  Ui

Each of the above quarter turn moves is a term, represented by a term ID or a term name. As with any set of terms, some of the terms in a Rubik’s Cube algorithm can be combined or redacted to simplify an algebraic sentence.

Use the space below to write down more relationships between Rubik’s terms and algebraic terms.

- Inverse Operations are operations that are opposite of each other and cancel each other out
- Inverted moves, such as Ri, cancels out an R move when done consecutively.

Teachers:
Algorithm and Term are the two most prevalent terms in this workbook; however, there are several terms that can be added.

You may wish to add terms yourself, or have the students keep a log of terms they use, on paper or mentally, while completing this project.

“Like terms” cannot simply be combined as there is an order to their operation. "Like term" in this case would need to result in the same outcome. Likewise, redacted terms are those that would cancel out another algorithm, therefore making it unnecessary. Students may choose to use trail and error techniques to discover which terms or moves are alike.

Teachers: Students are encouraged to note anything they may notice here in order to build mathematics confidence. All reasonable answers can be accepted.
Here, in the portion of your adventure, you must identify the variables, or the pieces of the solution, that represent the foundation of your goal as you move forward.

For many students, being able to solve the Rubik’s Cube is only the beginning. Many practice solving the Rubik’s Cube until they no longer need the algorithm guides. Some continue to solve the cube until they can do it in a certain amount of time or solve it faster than a friend.

You have learned the You CAN Do The Cube (YCDTC) method to solve The Rubik’s Cube. This method is sometimes referred to as the layer by layer method. There are 43 quintillion possible permutations to solve The Rubik’s Cube and many different solving methods. Your goal is to find a faster and/or more efficient way to solve the cube using the algorithms in the YCDTC SOLUTION GUIDE.

Once you become very familiar with the cube, you will notice there are some terms in the algorithms that are not necessary if you know the upcoming terms for the subsequent stages. Similarly, there are some goals that can be achieved by combining one term with another.

So, what is the or dilemma that needs a solution?

In order to find a more efficient way to solve the cube using the YCDTC method, the layer by layer method, I have to find the least amount of permutations. The problem is that there 43 quintillion possible permutations.

What is a possible solution?

I will need to figure out if there are some permutations that are not necessary or that can be combined with another move to solve the Rubik’s Cube the fastest way possible.
Recall the Modeling Cycle from the beginning of this workbook:

1. Problem
2. Formulate
3. Validate
4. Compute
5. Interpret
6. Report

Now that you have identified the problem, it is time to formulate the solution.

Be sure that your cube has been scrambled sufficiently, then beginning with Stage 2 and working through Stage 6 of the Solution Guide, you will write down every move, using its term ID. You will repeat the process at least three times.

Using the Modeling Cycle to formulate algebraic and statistical representations in order to understand the relationships between the data and the problem will help you create a more efficient algorithmic solution.

Use the following charts to collect your data. The data obtained while solving the Rubik's Cube three times will be used to formulate, potentially, more efficient solutions for the Rubik's Cube. As you are going through this exercise, make sure you note any observations such as turning the cube as a whole between algorithms or during an algorithm.

Teachers:

1. Term IDs are the names of the moves, such as R, Ri, L, Li.
2. Students may wish to create their own data collection table. Any recording system that reasonably collects and organizes the data efficiently will work. Students may also find more information than what is listed in the tables included in this book.
<table>
<thead>
<tr>
<th>Stage</th>
<th>Write every move in sequential order</th>
<th>Number of moves</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage 2</td>
<td>Ri, Di, Di, R, R, Ri, U, Fi, Ul, Di, R, R, Ri, U, Fi, Ul, Ri, Di, R, R</td>
<td>21</td>
</tr>
<tr>
<td>Stage 4</td>
<td>U, Ui, Li, U, L, U, F, Ut, Fi, U, U, U, R, Ut, Ri, Ut, Fi, U, F, Ut, U, R, Ut, Ri, Ut, Fi, U, F, U, R, Ut, Ri, Ut, Fi, U, F, U, R, Ut, Ri, Ut, Fi, U, F, U, R, Ut, Ri, Ut, Fi, U, F</td>
<td>74</td>
</tr>
</tbody>
</table>

Total number of moves: 215
### Data Collection 2

<table>
<thead>
<tr>
<th>Stage</th>
<th>Write every move in sequential order</th>
<th>Number of moves</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage 3</td>
<td>Ri, Di, R, D, D, D, Di, Ri, D, D, D, Di, Ri, D, D, Di, Ri, D, D, Di</td>
<td>18</td>
</tr>
</tbody>
</table>

**Total number of moves:** 131
## Data Collection 3

<table>
<thead>
<tr>
<th>Stage</th>
<th>Write every move in sequential order</th>
<th>Number # of moves</th>
</tr>
</thead>
</table>

**Total Number of moves:** 209
The next step in the Modeling Cycle will be to transition from **Formulate** to **Compute**.

To do this, you will need to analyze the data collected from the tables, which will ultimately guide you to the end goal of this project.

There are approximately, 43 quintillion algorithmic solutions to the Rubik’s Cube. You have identified three of them. To analyze the number of moves and their potential algorithm changes, find the mean of the **total number of moves** from the three data collection tables.

Mean = \( \frac{\text{sum of numbers}}{\text{quantity of addends}} \)

What is the mean, or the average, of moves to solve the Rubik’s Cube? (Use the space provided to show your work.)

\[
\text{Mean} = \frac{215 + 131 + 209}{3} = \frac{555}{3} = 185
\]

The average number of moves to solve the Rubik’s Cube with the **YCDTC method** is 185 moves.

What is 10% of the mean? (Use the space provided to show your work.)

Ten Percent of 185 is what?
\[
.10 \cdot 185 = x
\]
18.5 = x

Ten percent less moves would equal 167 moves.

10% less than 185 would be approx. 167 moves.

What is 25% of the mean? (Use the space provided to show your work.)

Twenty-five Percent of 185 is what?
\[
.25 \cdot 185 = x
\]
46.25 = x

Twenty-five percent less moves would equal 139 moves.

25% less than 185 would be approx. 139 moves.

**GOAL:**

Your goal is to develop a series of algorithms for each stage that have between 10% and 25% less moves than the mean you previously calculated.
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Teachers:

1. From CCSS.Math Practice.MP8.

Teachers:

2. If the solution does not work in the VALIDATE stage, students will need to revisit the FORMULATE or COMPUTE stages in order to find a better solution for the problem.

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Next, you will need to solve the Rubik’s Cube several times, paying close attention to the relationships between each term (move), as well as the relationship between the algorithms in each stage.

The charts below will help you to maintain oversight of the process, while attending to the details. It is imperative that you re-evaluate the reasonableness after each stage and any newly drawn conclusions. To validate, you need to continuously re-evaluate. If you are unable to validate your findings, return to the Formulate step in the Modeling Cycle.

While these charts have been designed to help you, you may have an idea that works better for you. Use any note-taking style that works for you. The goal is to find repeated, unnecessary terms/moves between stage transitions. It is possible that you will find terms within an algorithm that can be skipped or combined with another algorithm later. In between each of the Compute and Interpret steps in the cycle, you will be interpreting your results. You will not see the Interpret symbol of the Modeling Cycle throughout Part 2 as it is embedded in your process.

The terms/moves in each stage are noted on the following pages four times each. Each time you complete the algorithm, identify any terms that can be combined or eliminated and note them in the table. You should be able to repeat your findings at least three times before moving on.

Teachers:

If the solution does not work in the VALIDATE stage, students will need to revisit the FORMULATE or COMPUTE stages in order to find a better solution for the problem.
STAGE 2 ALGORITHM

Start by solving the Rubik’s Cube, up through Stage 2, the WHITE cross, from a scrambled state. The Solution Guide instructs you to keep the WHITE center piece on the UP face and to move a WHITE edge piece directly below its intended position. While using the following algorithm in Stage 2, identify any terms (moves) that can be combined or are unnecessary. Make notes on the table as needed. Repeat these instructions until you have achieved the same conclusion at least three times.

1. \( R R \) Then, to flip the white edge on top layer, if necessary. \( Ri \) U Fi Ui
2. \( R R \) Then, to flip the white edge on top layer, if necessary. \( Ri \) U Fi Ui
3. \( R R \) Then, to flip the white edge on top layer, if necessary. \( Ri \) U Fi Ui
4. \( R R \) Then, to flip the white edge on top layer, if necessary. \( Ri \) U Fi Ui

What conclusions did you make?

The R and Ri cancel each other out when used together.
Sometimes R, R puts the edge in the correct position and the algorithm is not needed.
The colors do not need to be completed in a specific order; you have to look at the placement of the white face; you might find an edge that is easier to place that will save you from doing extra moves.

Write your proposed algorithm: \( R R \) or \( R U Fi Ui \) or \( Ri U Fi Ui \)

What important instructions need to be communicated and followed to use your new algorithm? For example, “do this sequence 1, 2, or 3 times” and/or explain how the cube is positioned with reference to the faces, colors, corners.

Illustrate, if necessary. A blank cube has been provided should you need it.

If the white/color edge is matched correctly and directly below its intended position, use R R to position correctly.
If the white/color edge is matched incorrectly and directly below its intended position, use R U Fi Ui to position correctly.
If the white/color edge is matched incorrectly but positioned in the correct place on the top layer of the cube, use Ri U Fi Ui to position correctly.
**STAGE 3 ALGORITHM**

Start from a complete Stage 2 position. Solve the cube through Stage 3, the WHITE corners. The Solution Guide instructs you to keep the WHITE center piece on the UP face and to move a WHITE corner piece directly below its intended position. While using the following algorithm in Stage 3, identify any terms (moves) that can be combined or are unnecessary. Make notes on the table as needed. Repeat these instructions until you have achieved the same conclusion at least three times.

<table>
<thead>
<tr>
<th></th>
<th>To move the white corner from the top to the bottom layer</th>
<th>Ri</th>
<th>Di</th>
<th>R</th>
<th>then</th>
<th>Ri</th>
<th>Di</th>
<th>R</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td></td>
<td>Ri</td>
<td>Di</td>
<td>R</td>
<td>then</td>
<td>Ri</td>
<td>Di</td>
<td>R</td>
<td>D</td>
</tr>
<tr>
<td>2.</td>
<td></td>
<td>Ri</td>
<td>Di</td>
<td>R</td>
<td>then</td>
<td>Ri</td>
<td>Di</td>
<td>R</td>
<td>D</td>
</tr>
<tr>
<td>3.</td>
<td></td>
<td>Ri</td>
<td>Di</td>
<td>R</td>
<td>then</td>
<td>Ri</td>
<td>Di</td>
<td>R</td>
<td>D</td>
</tr>
<tr>
<td>4.</td>
<td></td>
<td>Ri</td>
<td>Di</td>
<td>R</td>
<td>then</td>
<td>Ri</td>
<td>Di</td>
<td>R</td>
<td>D</td>
</tr>
</tbody>
</table>

What conclusions did you make?

The number of times you do the algorithm depends on the position of the white face on the corner piece.

You can truncate the D move the last time the algorithm is done.

Write your proposed algorithm: **Ri, Di, R**. 

If you truncate the final D move after the white corner is correctly placed, you can save one move because your cube might be closer to the next desired position if you leave the D move out.

What important instructions need to be communicated and followed to use your new algorithm? For example, “do this sequence 1, 2, or 3 times?” and/or explain how the cube is positioned with reference to the faces, colors, corners.

Illustrate, if necessary. A blank cube has been provided should you need it.
Part 2b

An Algebraic Adventure

Before moving on to Stage 4, write down your conclusions from Stage 2 and 3. Write down your “new and improved,” more efficient algorithms.

Stage 2 - R, R or Ri, U, Fi, Ui or R, U, Fi, Ui
Stage 3 - Ri, Di, R, D except no D after last algorithm.

Test your new algorithms from Stage 2 and Stage 3 together several times. You will be validating after each stage to be sure all changes you have made work together with no errors.

Do the algorithms work consistently? Enter your answer on the chart below.

<table>
<thead>
<tr>
<th>Attempt</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
<th>6th</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

If you answered no, for any of the attempts, explain why your algorithm did not work.

If your test is unsuccessful, you need to return to the Compute step in the Modeling Cycle.

What notes would you include in the Solution Guide to help someone understand your more efficient algorithm suggestions?

Nothing different than what is in the guide already.

REMINDER:
The goal is to find repeated, unnecessary terms/moves between stage transitions that have between 10% and 25% less moves than the mean you previously calculated in Part 2a or a more efficient way to use the current algorithm.

It is possible that you will find terms within an algorithm that can be skipped or combined with another algorithm later.


**STAGE 4 ALGORITHM - CLOCKWISE**

Start from a complete Stage 3 position. Solve the cube through Stage 4, the MIDDLE layer. While using the following algorithm in Stage 4, identify any terms (moves) that can be combined or are unnecessary. Make notes on the table as needed. Repeat these instructions until you have achieved the same conclusion at least three times.

<table>
<thead>
<tr>
<th></th>
<th>R</th>
<th>Ui</th>
<th>Ri</th>
<th>Ui</th>
<th>Fi</th>
<th>U</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

What conclusions did you make?

Instead of vertically lining up the middle faces before doing the algorithm, you can align the vertical middle line with the bottom and middle layer on the front face and the top layer of the same color on the back face and do a Ui move instead of a U move. It eliminates 2 moves. Instead of doing a possible Ui, Vi or V, Vi, to align to the front face then Vi to align to the L face, you only have to do a Vi to get the L face.

Write your proposed algorithm: "Ui, R, Ui, Ri, Ui, Fi, U, F when aligned on the back face."

What important instructions need to be communicated and followed to use your new algorithm? For example, “do this sequence 1, 2, or 3 times?” and/or explain how the cube is positioned with reference to the faces, colors, corners.

Illustrate, if necessary. A blank cube has been provided should you need it.
**STAGE 4 ALGORITHM - COUNTER-CLOCKWISE**

Start from a complete Stage 3 position. Solve the cube through Stage 4, the MIDDLE layer. While using the following algorithm in Stage 4, identify any terms (moves) that can be combined or are unnecessary. Make notes on the table as needed. Repeat these instructions until you have achieved the same conclusion at least three times.

1. \( U_i U L U F U_i F_i \)
2. \( U_i U L U F U_i F_i \)
3. \( U_i U L U F U_i F_i \)
4. \( U_i U L U F U_i F_i \)

What conclusions did you make?

Instead of vertically lining up the middle faces before doing the algorithm, you can align the vertical middle line with the bottom and middle layer on the front face and the top layer of the same color on the back face and do a \( U \) move instead of a \( U_i \) move. It eliminates 2 moves. Instead of doing a possible \( U_i, U_i \) or \( U, U \) to get to the \( R \) face.

Write your proposed algorithm: \( U, R, U_i, R_i, U_i, F_i, U, F \) when aligned on the back face.

What important instructions need to be communicated and followed to use your new algorithm? For example, “do this sequence 1, 2, or 3 times?” and/or explain how the cube is positioned with reference to the faces, colors, corners.

Illustrate, if necessary. A blank cube has been provided should you need it.
Before moving on to Stage 5, validate your conclusions from Stage 3 and 4.

Write down your “new and improved,” more efficient algorithms.

Stage 3 - Ri, Di, R, D except no D after last algorithm.
Stage 4 - Clockwise - U, R, U, Ri, U, Fi, U, F
               when aligned on the back face.
               Counter-Clockwise - U, R, U, Ri, U, Fi, U, F
               when aligned on the back face.

Test your new algorithms from Stage 3 and Stage 4 together several times. You will be validating after each stage to be sure all changes you have made work together with no errors.

Do they work every time? Enter your answer on the chart below.

<table>
<thead>
<tr>
<th>1st Attempt</th>
<th>2nd Attempt</th>
<th>3rd Attempt</th>
<th>4th Attempt</th>
<th>5th Attempt</th>
<th>6th Attempt</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
</tbody>
</table>

If you answered no, for any of the attempts, explain why your algorithm did not work.

The scenario did not present during those attempts.

If your test is unsuccessful, you need to return to the Compute step in the Modeling Cycle.

What notes would you include in the Solution Guide to help someone understand your more efficient algorithm suggestions:

This works consistently when the middle face of the top layer naturally ends up on the back face. If you purposefully line it up, then you might add extra moves. If the middle face of the top layer is already on the L or R face, you might not need any U or Ui turns and can start with R.

Teachers: Validating work is key, both for creating new algorithms for the Rubik’s Cube and for checking one’s work in Algebra. The purpose of using “Validate” repeatedly is to encourage students to build a habit after each step in addition to an overall check.

While continuous validating may seem redundant to students, they should be reminded that should they only validate at the end, it would be more difficult to identify the error and redo all subsequent stages post error.

Reminder: It is imperative that you re-evaluate the reasonableness after each stage and any newly drawn conclusions. To validate, you need to continuously re-evaluate.
**STAGE 5 has two steps.**

**STAGE 5a ALGORITHM**

Within STAGE 5a, evaluate each algorithm with its respective starting position. Refer to Page 6 of the Solution Guide.

<table>
<thead>
<tr>
<th>State 1</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>State 2</th>
<th>F</th>
<th>U</th>
<th>R</th>
<th>Ui</th>
<th>Ri</th>
<th>Fi</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>State 3</th>
<th>F</th>
<th>U</th>
<th>R</th>
<th>Ui</th>
<th>Ri</th>
<th>Fi</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>State 4</th>
<th>F</th>
<th>R</th>
<th>U</th>
<th>Ri</th>
<th>Ui</th>
<th>Fi</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

States 2 and 3 use the same algorithm. Why?

States 2 and 3 use the same algorithm because the F, U, R, Ui, Ri, Fi, algorithm gets the cube to the State 4 image.

State 4 uses a different algorithm. Why is it different from States 2 and 3? What is the significance of the differences?

State 4 algorithm is the opposite of States 2 and 3.

If you don't have state 1, then you want to get to State 4.
If you get to State 4 and do the algorithm F, R, U, Ri, Ui, Fi then it only takes 1 algorithm to get to State 1.

The tables on the following pages will help you identify whether or not there is one common algorithm that can be applied to all three states.

---

**Teachers:**

If students are having an issue with solving State 4, have them check to be sure they are using the correct algorithm. State 4 is different than States 2 and 3.
### An Algebraic Adventure

#### STAGE 5a ALGORITHM FOR STATES 2 & 3

Start from a complete Stage 4 position. Solve the cube through Stage 5a, the YELLOW cross. Identify whether or not the given algorithm works for each of the states pictured at the left of the table.

<table>
<thead>
<tr>
<th>State</th>
<th>Algorithm</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>F U R Ui Ri Fi</td>
<td>Yes</td>
</tr>
<tr>
<td>3</td>
<td>F U R Ui Ri Fi</td>
<td>Yes</td>
</tr>
<tr>
<td>4</td>
<td>F U R Ui Ri Fi</td>
<td>No</td>
</tr>
</tbody>
</table>

#### STAGE 5a ALGORITHM FOR STATE 4

Start from a complete Stage 4 position. Solve the cube through Stage 5a, the YELLOW cross. Identify whether or not the given algorithm works for each of the states pictured at the left of the table.

<table>
<thead>
<tr>
<th>State</th>
<th>Algorithm</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>F R U Ri Ui Fi</td>
<td>Yes if done 3x</td>
</tr>
<tr>
<td>3</td>
<td>F R U Ri Ui Fi</td>
<td>Yes if done 2x</td>
</tr>
<tr>
<td>4</td>
<td>F R U Ri Ui Fi</td>
<td>Yes</td>
</tr>
</tbody>
</table>

---

**Teachers:**

In this "test," the same algorithm is being used for each state. It is important that this test not be confused with the given algorithms on Page 25, or in the Solution Guide.
STAGE 5a FIRST ALGORITHM POSSIBILITY

Write your proposed algorithm: F, R, U, R, U, F

Start from a complete Stage 4 position. Solve the cube through Stage 5a, the YELLOW cross using your proposed algorithm from above. Identify whether or not your proposed algorithm works for each of the states pictured at the left of the table. Repeat your tests until you have achieved the same conclusion at least three times for each state. Remember, your goal is to have the YELLOW cross on the UP face.

Write yes or no in the boxes given to indicate whether or not the algorithm worked to achieve the YELLOW Cross.

<table>
<thead>
<tr>
<th>State 2</th>
<th>1st Attempt</th>
<th>2nd Attempt</th>
<th>3rd Attempt</th>
<th>4th Attempt</th>
<th>5th Attempt</th>
<th>6th Attempt</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No</td>
<td>No</td>
<td>Yes, 1x to get State 3</td>
<td>Yes, 1x to get State 1</td>
<td>Yes, 1x to get State 3</td>
<td>Yes, 1x to get State 1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>State 3</th>
<th>1st Attempt</th>
<th>2nd Attempt</th>
<th>3rd Attempt</th>
<th>4th Attempt</th>
<th>5th Attempt</th>
<th>6th Attempt</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Yes, 1x to get State 1</td>
<td>Yes, 1x to get State 3</td>
<td>Yes, 1x to get State 1</td>
<td>Yes, 1x to get State 3</td>
<td>Yes, 1x to get State 1</td>
<td>Yes, 1x to get State 1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>State 4</th>
<th>1st Attempt</th>
<th>2nd Attempt</th>
<th>3rd Attempt</th>
<th>4th Attempt</th>
<th>5th Attempt</th>
<th>6th Attempt</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Yes, 1x to get State 1</td>
<td>Yes, 1x to get State 1</td>
<td>Yes, 1x to get State 1</td>
<td>Yes, 1x to get State 1</td>
<td>Yes, 1x to get State 1</td>
<td>Yes, 1x to get State 1</td>
</tr>
</tbody>
</table>

If your first algorithm was unsuccessful, try again. Create another algorithm and use the follow chart to test it. Remember, there are 43 quintillion permutation possibilities to solve the Rubik’s Cube. Be patient and keep trying.
**An Algebraic Adventure**

**STAGE 5a SECOND ALGORITHM POSSIBILITY**

Write your proposed algorithm: **N/A**

Start from a complete Stage 4 position. Solve the cube through Stage 5a, the YELLOW cross using your proposed algorithm from above. Identify whether or not your proposed algorithm works for each of the states pictured at the left of the table. Repeat your tests until you have achieved the same conclusion at least three times for each state. Remember, your goal is to have the YELLOW cross on the UP face.

Write yes or no in the boxes given to indicate whether or not the algorithm worked to achieve the YELLOW cross.

<table>
<thead>
<tr>
<th>State 2</th>
<th>1st Attempt</th>
<th>2nd Attempt</th>
<th>3rd Attempt</th>
<th>4th Attempt</th>
<th>5th Attempt</th>
<th>6th Attempt</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Already successful with first algorithm.</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>State 3</th>
<th>1st Attempt</th>
<th>2nd Attempt</th>
<th>3rd Attempt</th>
<th>4th Attempt</th>
<th>5th Attempt</th>
<th>6th Attempt</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>State 4</th>
<th>1st Attempt</th>
<th>2nd Attempt</th>
<th>3rd Attempt</th>
<th>4th Attempt</th>
<th>5th Attempt</th>
<th>6th Attempt</th>
</tr>
</thead>
</table>

**STAGE 5a THIRD ALGORITHM POSSIBILITY (IF NEEDED)**

Write your proposed algorithm: **N/A**

<table>
<thead>
<tr>
<th>State 2</th>
<th>1st Attempt</th>
<th>2nd Attempt</th>
<th>3rd Attempt</th>
<th>4th Attempt</th>
<th>5th Attempt</th>
<th>6th Attempt</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>State 3</th>
<th>1st Attempt</th>
<th>2nd Attempt</th>
<th>3rd Attempt</th>
<th>4th Attempt</th>
<th>5th Attempt</th>
<th>6th Attempt</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>State 4</th>
<th>1st Attempt</th>
<th>2nd Attempt</th>
<th>3rd Attempt</th>
<th>4th Attempt</th>
<th>5th Attempt</th>
<th>6th Attempt</th>
</tr>
</thead>
</table>
STAGE 5a ALGORITHM

What important instructions need to be communicated and followed to use your new algorithm? For example, “do this sequence 1, 2, or 3 times?” and/or explain how the cube is positioned with reference to the faces, colors, corners.

Illustrate, if necessary. A blank cube has been provided should you need it.

You can use the same algorithm for each image state, but after each algorithm, you have to rematch the cube to the next state.

If you have to complete the algorithm more than once without rematching the image, you can truncate the last move $F_i$ on the first algorithm and the first move, $F$, on the second algorithm because they cancel each other out.
An Algebraic Adventure

Before moving on to Stage 5b, write down your conclusions from Stage 4 and 5.

Write down your altered, or "new and improved," more efficient algorithms.

Stage 4 - Clockwise U, R, Uı, Rı, Uı, Fi, U, F
when aligned on the back face.
Counter-Clockwise U, R, Uı, Rı, Uı, Fi, U, F
when aligned on the back face.

Stage 5a - F, R, Uı, Rı, Uı, Fi

Test your new algorithms from Stage 4 and Stage 5 together several times. You will be validating after each stage to be sure all changes you have made work together with no errors.

Do they work every time? Enter your answer on the chart below.

<table>
<thead>
<tr>
<th>1st Attempt</th>
<th>2nd Attempt</th>
<th>3rd Attempt</th>
<th>4th Attempt</th>
<th>5th Attempt</th>
<th>6th Attempt</th>
</tr>
</thead>
<tbody>
<tr>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

If you answered no, for any of the attempts, explain why your algorithm did not work.

The scenario did not present during first attempt.

If your test is unsuccessful, you need to return to the Compute step in the Modeling Cycle.

What notes would you include in the Solution Guide to help someone understand your more efficient algorithm suggestions:

The algorithms are dependent on the position of the individual cubes, faces, edges, corners, etc., just like in the original Solution Guide.
### STAGE 5b ALGORITHM

In STAGE 5b there is one common algorithm that can be applied to all three states. Given that there are three states, the assumption is that one state is more efficient in getting to the 5b goal, the YELLOW face. Is there a specific state that is more efficient in achieving a completed Stage 5b faster? Are there any combinations or redactions in the algorithm to make the algorithm more efficient? In the following pages, you will develop and test the given algorithm to answer these questions.

<table>
<thead>
<tr>
<th>State 1</th>
<th>R</th>
<th>U</th>
<th>Ri</th>
<th>U</th>
<th>R</th>
<th>U</th>
<th>U</th>
<th>Ri</th>
</tr>
</thead>
<tbody>
<tr>
<td>State 2</td>
<td>R</td>
<td>U</td>
<td>Ri</td>
<td>U</td>
<td>R</td>
<td>U</td>
<td>U</td>
<td>Ri</td>
</tr>
<tr>
<td>State 3</td>
<td>R</td>
<td>U</td>
<td>Ri</td>
<td>U</td>
<td>R</td>
<td>U</td>
<td>U</td>
<td>Ri</td>
</tr>
</tbody>
</table>

Things to consider when testing the algorithm:

- The UP face of States 1, 2, and 3 are the same. They each show the YELLOW cross.
- Is it necessary to have the YELLOW piece always in the LEFT corner like in one of the three states shown?
- How many times do you need to to the algorithm from each state to complete the Yellow Corners?

Possible thought process to consider:

1. Formulate
2. Compute
3. Validate
   possible solutions.
STAGE 5b FIRST ALGORITHM POSSIBILITY

Use the following chart to test the algorithm. Repeat your test until you have achieved the same conclusion at least three times.

Write your proposed algorithm: \( R, U, Ri, U, R, U, U, Ri \) one time.

Start from a complete Stage 5a position. Solve the cube through Stage 5b, the YELLOW corners using your proposed algorithm from above. Identify whether or not your proposed algorithm works for each of the states pictured at the left of the table. Repeat your tests until you have achieved the same conclusion at least three times for each state. Remember, your goal is to have the YELLOW corners on the UP face.

Write yes or no in the boxes given to indicate whether or not the algorithm worked to achieve the YELLOW corners. If you answer no, include a brief explanation.

<table>
<thead>
<tr>
<th>State</th>
<th>1st Attempt</th>
<th>2nd Attempt</th>
<th>3rd Attempt</th>
<th>4th Attempt</th>
<th>5th Attempt</th>
<th>6th Attempt</th>
</tr>
</thead>
<tbody>
<tr>
<td>State 1</td>
<td>No, But, it gets me to state 2</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>State 2</td>
<td>Yes, this gets me to my goal. This is the best state to get to the next stage fastest.</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>State 3</td>
<td>No, But, it gets me to state 2</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

If your first algorithm was unsuccessful, try again. Remember, there are 43 quintillion permutation possibilities to solve the Rubik’s Cube. Be patient and keep trying.
**STAGE 5b SECOND ALGORITHM POSSIBILITY**

Use the following chart to test the possible algorithm. Repeat your test until you have achieved the same conclusion at least three times.

\[ Ru, U, Ri, U, Ru, U, Ru, U \text{ or } \]

Write your proposed algorithm: \( Ru, U, Ri, U, Ru, U, Ru, U \text{ or } \)

Start from a complete Stage 5a position. Solve the cube through Stage 5b, the YELLOW corners using your proposed algorithm from above. Identify whether or not your proposed algorithm works for each of the states pictured at the left of the table. Repeat your tests until you have achieved the same conclusion at least three times for each state. Remember, your goal is to have the YELLOW corners on the UP face.

Write yes or no in the boxes given to indicate whether or not the algorithm worked to achieve the YELLOW corners. If you answer no, include a brief explanation.

<table>
<thead>
<tr>
<th>State 1</th>
<th>1st Attempt</th>
<th>2nd Attempt</th>
<th>3rd Attempt</th>
<th>4th Attempt</th>
<th>5th Attempt</th>
<th>6th Attempt</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>x2 YES</td>
<td>x2 YES</td>
<td>x2 YES</td>
<td>x2 YES</td>
<td>x2 YES</td>
<td>x2 YES</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>State 2</th>
<th>1st Attempt</th>
<th>2nd Attempt</th>
<th>3rd Attempt</th>
<th>4th Attempt</th>
<th>5th Attempt</th>
<th>6th Attempt</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>x2 YES</td>
<td>x2 No</td>
<td>x2 No</td>
<td>x2 YES</td>
<td>x2 No</td>
<td>x2 Yes</td>
</tr>
</tbody>
</table>

*It worked when the yellow corner was on the front face

<table>
<thead>
<tr>
<th>State 3</th>
<th>1st Attempt</th>
<th>2nd Attempt</th>
<th>3rd Attempt</th>
<th>4th Attempt</th>
<th>5th Attempt</th>
<th>6th Attempt</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>x2 YES</td>
<td>x2 YES</td>
<td>x2 YES</td>
<td>x2 YES</td>
<td>x2 YES</td>
<td>x2 Yes</td>
</tr>
</tbody>
</table>

**STAGE 5b THIRD ALGORITHM POSSIBILITY (IF NEEDED)**

Write your proposed algorithm: \( Ru, U, Ri, U, Ru, U, Ru, U, Ri \) with a solved corner on the front/left without rematching the cube

<table>
<thead>
<tr>
<th>State 1</th>
<th>1st Attempt</th>
<th>2nd Attempt</th>
<th>3rd Attempt</th>
<th>4th Attempt</th>
<th>5th Attempt</th>
<th>6th Attempt</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>State 2</th>
<th>1st Attempt</th>
<th>2nd Attempt</th>
<th>3rd Attempt</th>
<th>4th Attempt</th>
<th>5th Attempt</th>
<th>6th Attempt</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>State 3</th>
<th>1st Attempt</th>
<th>2nd Attempt</th>
<th>3rd Attempt</th>
<th>4th Attempt</th>
<th>5th Attempt</th>
<th>6th Attempt</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>
The goal is to get to state 2 because from state 2, it only takes one time of the algorithm to get to the goal if the yellow corner is on the front face on the right layer.

If you don’t have to reorient the cube in between states, you can combine Ri and R because they cancel each other out.
Before moving on to Stage 6, write down your conclusions from Stage 5a and 5b.

Write down your altered, or "new and improved," more efficient method.

**Stage 5a - F, R, U, Ri, Ul, Fx**

**Stage 5b - R, U, Ri, U, R, U, U, Ri**

Sometimes: R, U, Ri, U, R, U, U (U, Ri, U, R U, U) Ri

Test your new algorithms from Stage 5a and Stage 5b together several times. You will be validating after each stage to be sure all changes you have made work together with no errors.

Do they work every time? Enter your answer on the chart below.

<table>
<thead>
<tr>
<th>Attempt</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
<th>6th</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

If you answered no, for any of the attempts, explain why your algorithm did not work.

If your test is unsuccessful, you need to return to the Compute step in the Modeling Cycle.

What notes would you include in the Solution Guide to help someone understand your more efficient algorithm suggestions:

The algorithms are dependent on the position of the individual cubes, edges, corners, etc., just like in the original Solution Guide.
An Algebraic Adventure

Remember, the goal is to find repeated, unnecessary terms/moves between stage transitions or a more efficient way to use the current algorithm. It is possible that you will find terms within an algorithm that can be skipped or combined with another algorithm later.

The terms/moves in each stage are noted four times each. Each time you complete the algorithm, identify any terms that can be combined or eliminated and note them in the table. You should be able to repeat your findings at least three times before moving on.

### STAGE 6a ALGORITHM

Start from a complete Stage 5b position. Solve the cube through Stage 6a, the YELLOW face. While using the following algorithm in Stage 6a, identify any terms (moves) that can be combined or are unnecessary. Make notes on the table as needed. Repeat these instructions until you have achieved the same conclusion at least three times.

| 1. | Ri | F | Ri | B | B | R | Fi | Ri | B | B | R | R | Ui |
|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 2. | Ri | F | Ri | B | B | R | Fi | Ri | B | B | R | R | Ui |
| 3. | Ri | F | Ri | B | B | R | Fi | Ri | B | B | R | R | Ui |
| 4. | Ri | F | Ri | B | B | R | Fi | Ri | B | B | R | R | Ui |

What conclusions did you make?

I found that if this step is needed, then it’s the best algorithm for this step. However, I did conclude that this step is not always needed. Sometimes this algorithm can be eliminated altogether. It depends on the end position of Stage 5b.

I am curious to know if there’s a way to solve Stage 5b so that 6a is never needed.

Write your proposed algorithm: N/A

What important instructions need to be communicated and followed to use your new algorithm? For example, “do this sequence 1, 2, or 3 times” and/or explain how the cube is positioned with reference to the faces, colors, corners.

Illustrate, if necessary. A blank cube has been provided should you need it.

This step may be skipped if the top layer is already in position to solve Stage 6b.
Part 2b

An Algebraic Adventure

Before moving on to Stage 6b, write down your conclusions from Stage 5a, 5b, and 6a.

Write down the altered or, “new and improved,” more efficient algorithms.

**Stage 5a** - F, R, U, Ri, Ui, Fi

**Stage 5b** - R, U, Ri, U, R, U, U, Ri


**Stage 6a** - Ri, F, Ri, B, B, R, Fi, Ri, B, B, R, R, Ui

Test your new algorithms from Stage 5a, 5b and Stage 6a together several times. You will be validating after each stage to be sure all changes you have made work together with no errors.

Do they work every time? Enter your answer on the chart below.

<table>
<thead>
<tr>
<th>1st Attempt</th>
<th>2nd Attempt</th>
<th>3rd Attempt</th>
<th>4th Attempt</th>
<th>5th Attempt</th>
<th>6th Attempt</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

If you answered no, for any of the attempts, explain why your algorithm did not work.

If your test is unsuccessful, you need to return to the Compute step in the Modeling Cycle.

What notes would you include in the Solution Guide to help someone understand your more efficient algorithm suggestions:
### STAGE 6b ALGORITHM (CLOCKWISE)

Start from a complete Stage 6a position. Solve the cube through Stage 6b, the YELLOW edges. While using the following algorithm in Stage 6b, identify any terms (moves) that can be combined or are unnecessary. Make notes on the table as needed. Repeat these instructions until you have achieved the same conclusion at least three times.

<table>
<thead>
<tr>
<th>Step</th>
<th>Moves</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>F F U L Ri F F Li R U F F</td>
</tr>
<tr>
<td>2.</td>
<td>F F U L Ri F F Li R U F F</td>
</tr>
<tr>
<td>3.</td>
<td>F F U L Ri F F Li R U F F</td>
</tr>
<tr>
<td>4.</td>
<td>F F U L Ri F F Li R U F F</td>
</tr>
</tbody>
</table>

What conclusions did you discover?

Sometimes you have to perform the algorithm twice to position the yellow edges correctly. If you have to do it twice without reorienting the cube, you can truncate the last two moves, F, F, and the first two moves, F, F, because F, F, F, F is a 360 degree turn and the front face ends exactly where it belongs.


What important instructions need to be communicated and followed to use your new algorithm? For example, “do this sequence 1, 2, or 3 times?” and/or explain how the cube is positioned with reference to the faces, colors, corners.

Illustrate, if necessary. A blank cube has been provided should you need it.

This is only a more efficient method if you have to perform the algorithm twice.
STAGE 6b ALGORITHM (COUNTER-CLOCKWISE)

Start from a complete Stage 6a position. Solve the cube through Stage 6b, the yellow edges. While using the following algorithm in Stage 6b, identify any terms (moves) that can be combined or are unnecessary. Make notes on the table as needed. Repeat these instructions until you have achieved the same conclusion at least three times.

1. F F Ui L Ri F F Li R Ui F F
2. F F Ui L Ri F F Li R Ui F F
3. F F Ui L Ri F F Li R Ui F F
4. F F Ui L Ri F F Li R Ui F F

What conclusions did you discover?

The only difference between clockwise and counter-clockwise algorithm is use of Ui instead of U.

Sometimes you have to perform the algorithm twice to position the yellow edges correctly. If you have to do it twice without reorienting the cube, you can truncate the last two moves, F, F, and the first two moves, F, F, because F, F, F is a 360 degree turn and the front face ends exactly where it belongs.


What important instructions need to be communicated and followed to use your new algorithm? For example, “do this sequence 1, 2, or 3 times?” and/or explain how the cube is positioned with reference to the faces, colors, corners.

Illustrate, if necessary. A blank cube has been provided should you need it.

This is only a more efficient method if you have to perform the algorithm twice.
An Algebraic Adventure

Before moving on to the remaining step, you must first validate your conclusions from STAGES 5a, 5b, 6a, and 6b.

Stage 5a - F, R, U, R, U, L, F
Stage 6b -

Test your new algorithms from Stage 5a, 5b 6a and Stage 6b together several times. You will be validating after each stage to be sure all changes you have made work together with no errors.

Do they work every time? Enter your answer on the chart below.

<table>
<thead>
<tr>
<th>1st Attempt</th>
<th>2nd Attempt</th>
<th>3rd Attempt</th>
<th>4th Attempt</th>
<th>5th Attempt</th>
<th>6th Attempt</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
</tbody>
</table>

If you answered no, for any of the attempts, explain why your algorithm did not work.

The scenario did not present during those attempts.

If your test is unsuccessful, you need to return to the Compute step in the Modeling Cycle.

What notes would you include in the Solution Guide to help someone understand your more efficient algorithm suggestions:
Use this page to write down the new algorithms for Stages 2 through 6, in a format similar to Page 10. Test your algorithms several times to ensure the Stages work when combined together. If you are able to solve the cube repeatedly with your altered algorithms, you are ready for the last section.

### White Cross (2):
- R R Or Ri, U, Fi, Ui Or R, U, Fi, Ui

### White Corners (3):
- Ri, Di, R, D
  - except no D after last algorithm.

### Middle Layer (4):
- **Clockwise:** U, (Ui), R, Ui, Ri, Ui, Fi, Ui, F
  - (when aligned on the back face)
- **Counter-Clockwise:** Ui, (U) Li, U, L, U, F, Ui, Fi
  - (when aligned on the back face)

### Yellow Cross (5a): F, R, U, Ri, Ui, Fi

### Yellow Corners (5b):
- R, U, Ri, U, R, U, U, Ri

### Yellow Face (6a):

### Yellow Edges (6b):
- **(if required 2 times)**
  - **Counter-Clockwise -** F, F, Ui, L, Ri, F, F, Li, R, U (Ui, L, Ri, F, F, Li, R, Ri, Ui, F, F)

### 1st Attempt 2nd Attempt 3rd Attempt 4th Attempt 5th Attempt 6th Attempt
- Yes  Yes  Yes  Yes  Yes  Yes
On page 17 of this workbook, you found the mean number of moves after solving the cube, and collecting data three times. Your goal, from that point, was to develop a faster, move efficient, way to solve the Rubik’s Cube by simplifying the algorithms. Now that you have formulated, computed, and validated your new and improved algorithms and instructions to the layer by layer solution method, it’s time to re- so you can compare and your new data.

Using the following charts, similar to the data collection charts on pages 14-16, tally the number of moves it takes to solve the Rubik’s Cube using your new and improved algorithms and instructions. You do not need to write down the name of each move; simply tally the number of moves.

<table>
<thead>
<tr>
<th>Stage 2</th>
<th>Tally the number of moves you perform for each stage</th>
<th>Number of moves</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>11111 1111 1111</td>
<td>14</td>
</tr>
<tr>
<td>Stage 3</td>
<td>11111 1111 1111 1111 1111 1111 1111 1111 1111 1111</td>
<td>42</td>
</tr>
<tr>
<td>Stage 4</td>
<td>11111 1111 1111 1111 1111 1111 1111 1111 1111 1111</td>
<td>22</td>
</tr>
<tr>
<td>Stage 5a &amp; 5b</td>
<td>5a - 11111 11111 1</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td>5b - 11111 11111 11111 11111 11111 11111 11111 11111 11111</td>
<td>56</td>
</tr>
</tbody>
</table>

Total number of moves: 169
### Data Collection 2

<table>
<thead>
<tr>
<th>Stage</th>
<th>Tally the number of moves you perform for each stage</th>
<th>Number of moves</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage 2</td>
<td>11111 11111 11111 11111 11111 11111</td>
<td>22</td>
</tr>
<tr>
<td>Stage 3</td>
<td>11111 11111 11111 11111 11111 11111 11111 11111 11111</td>
<td>56</td>
</tr>
<tr>
<td>Stage 4</td>
<td>11111 11111 11111 11111 11111 11111 11111 11111 11111 11111</td>
<td>30</td>
</tr>
<tr>
<td>Stage 5a &amp; 5b</td>
<td>11111 11111 11111 11111 11111 11111 11111 11111 11111 11111 11111 11111</td>
<td>16</td>
</tr>
<tr>
<td>Stage 6a &amp; 6b</td>
<td>11111 11111 11111 11111 11111 11111 11111 11111 11111 11111 11111 11111 11111</td>
<td>49</td>
</tr>
<tr>
<td></td>
<td><strong>Total number of moves:</strong></td>
<td><strong>172</strong></td>
</tr>
</tbody>
</table>

### Data Collection 3

<table>
<thead>
<tr>
<th>Stage</th>
<th>Tally the number of moves you perform for each stage</th>
<th>Number of moves</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stage 2</td>
<td>11111 11111 11111 11111 11111 11111 11111 11111 11111 11111</td>
<td>33</td>
</tr>
<tr>
<td>Stage 3</td>
<td>11111 11111 11111 11111 11111 11111 11111 11111 11111 11111</td>
<td>23</td>
</tr>
<tr>
<td>Stage 4</td>
<td>11111 11111 11111 11111 11111 11111 11111 11111 11111 11111</td>
<td>23</td>
</tr>
<tr>
<td>Stage 5a &amp; 5b</td>
<td>11111 11111 11111 11111 11111 11111 11111 11111 11111 11111</td>
<td>20</td>
</tr>
<tr>
<td>Stage 6a &amp; 6b</td>
<td>11111 11111 11111 11111 11111 11111 11111 11111 11111 11111</td>
<td>34</td>
</tr>
<tr>
<td></td>
<td><strong>Total number of moves:</strong></td>
<td><strong>133</strong></td>
</tr>
</tbody>
</table>
The next step in the Modeling Cycle will be to transition from ________ to ________.

To do this, you will need to analyze the data collected from the tables, which will ultimately guide you to the end goal of this project.

There are approximately, 43 quintillion algorithmic solutions to the Rubik’s Cube. You have modified the YCDTC layer by layer method to create a more efficient way to solve the Rubik’s Cube; you have validated your moves and collected new data for the number of moves needed to solve the cube from an arbitrary starting point on a scrambled cube.

In your original data collection, what was the average number of moves performed to solve the cube? (Page 17)

$185$

List the totals from the three data collection tables on pages 42 and 43 where you used your NEW algorithms.

\[
\begin{align*}
169 \\
172 \\
133
\end{align*}
\]

To analyze the number of moves and their potential algorithm changes, find the mean of the total number of moves from the three data collection tables.

\[
\text{Mean} = \frac{\text{sum of numbers}}{\text{quantity of addends}}
\]

What is the mean, or the average, number of moves to solve the Rubik’s Cube using the NEW algorithms?

\[
\text{Mean} = \frac{169 + 172 + 133}{3} = \frac{474}{3} = 158
\]

Your original goal was to develop a series of algorithms for each stage that result in 10% or 25% less moves than the mean you previously calculated.

What percentage less than the original moves were you able to accomplish?

\[
158 \text{ is what percent of } 185? \quad \frac{158}{185} \times 100 \approx 85\%
\]

I was able to reduce the average number of moves by 15%.
An Algebraic Adventure - Teacher Guide

An Algebraic Adventure

Using the following pages and blank paper if necessary, you are going to design your own SOLUTION GUIDE, with the algorithms, stages, states, and steps that you’ve created throughout this workbook. You will need to complete the instructions and algorithms for each stage, and design your own cover. Use the You CAN Do The Rubik’s Cube SOLUTION GUIDE to help guide you. If you choose to develop your SOLUTION GUIDE digitally, print out the pages and attach them to the following pages.

Use the space below to design your front page.

---

Teachers:
The next 8 pages (pages 46-53) included in the student workbook are just like the page below to enable the student to design their own Solution Guide.

However, should you want to display the solution guide your students create, you may want to encourage a more creative, or elaborate, design.
An Algebraic Adventure - Teacher Guide

An Algebraic Adventure

REFLECTIONS

What math concepts did you feel you used most often?

I mostly used like terms and algorithms. I did have to “undo” my moves sometimes. When I undo moves, it feels similar to using the inverse property. Some of the language I used to do this workbook is similar to language I use when I do algebra, such as algorithm and truncate. The modeling cycle is similar to how we solve problems in class. In class we don’t say validate we say check your answer or prove your work.

Describe relationships between faces of the cube and independent pieces within the cube.

The pieces of the cube either have 2 or 3 faces and depending on how you move them, the faces are in different positions. Even though the faces are in a different position, they still have the same value, just like in algebra; just because a term is in a different place or combined with something else doesn’t change it’s value.

What have you deduced about processes of reasoning? Repeated reasoning? Explaining reasoning?

Reasoning is a long process. I understand that repeated reasoning helps to prove the work consistently and helps so that I can predict an answer or a reason in another problem if I understand it in this problem. By explaining all my steps and conclusions, I had to really think about what words to use and how to say things clearly so someone else could understand. By doing this, I was able to find my own errors too.
REFLECTIONS

How does the Problem Solving Model help (or not help) you make sense of problematic situations?

The Problem Solving Model helped me organize my solutions. If I made a mistake, I knew how to go back and find it and find a solution for the mistake. In this workbook, the Problem Solving Model was used for the whole project, but I also used it for each step to help me stay organized and not make too many errors.

This adventure required perseverance. How can you use the qualities of perseverance when solving other problematic situations in math? In real-life?

There were many times I felt frustrated or that the work was too tedious, but I stuck with it and the more I did and the more mistakes I had to fix, the more I learned. It felt good to learn and be able to achieve something. I feel like I can apply this to the really difficult problems we do in class that take a long time to solve.